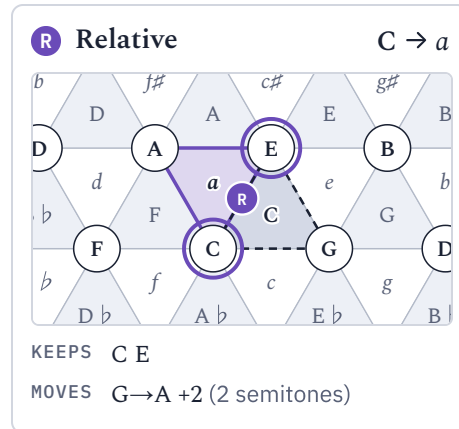
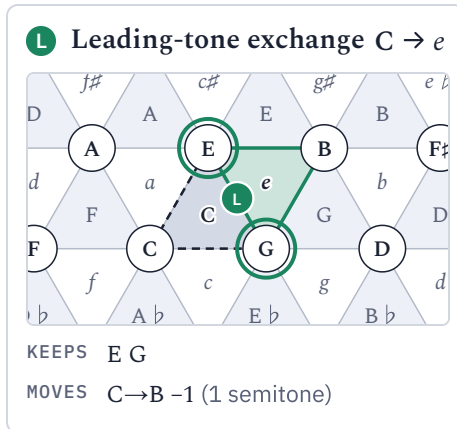
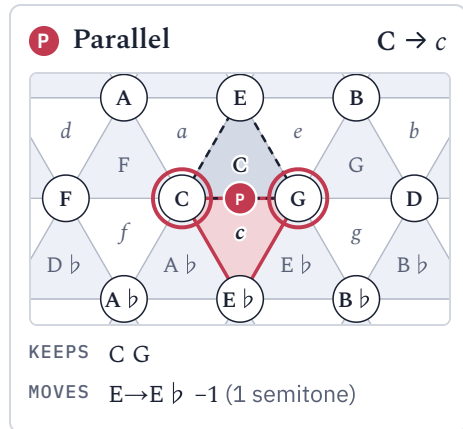


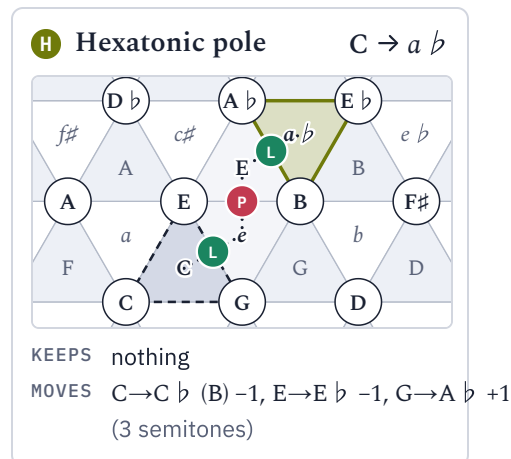
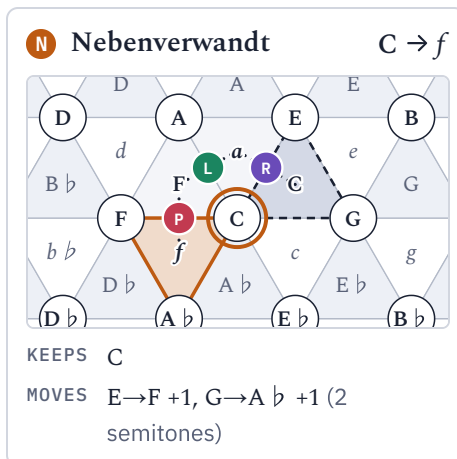
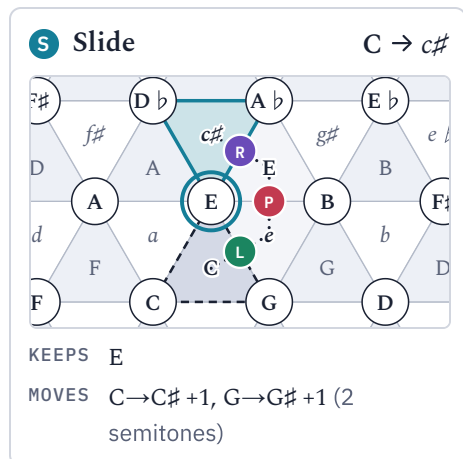
All six transformations

S, N and H are compound moves: chains of basic moves treated as one step. Each panel starts on C major (dashed) and lands on the shaded triangle. Ringed notes stay put, and the letters on the dotted line spell out the chain.

BASIC MOVES • ONE NOTE MOVES



COMPOUND MOVES • TWO OR THREE NOTES MOVE



Each compound move has two chains: S = LPR or RPL, N = RLP or PLR, H = LPL or PLP. David Lewin named the slide, and “hexatonic pole” is Richard Cohn’s term. N comes from Riemann’s theory and links major tonic to minor subdominant ($I \rightarrow iv$) or minor tonic to major dominant ($i \rightarrow V$).

Every triad, every move Find the starting triad on the left and read across. Capitals are major, lower case is minor.

MAJOR	P	L	R	S	N	H	MINOR	P	L	R	S	N	H
C	c	e	a	c#	f	a b	c	C	A b	E b	B	G	E
D b	c#	f	b b	d	f#	a	c#	D b	A	E	C	A b	F
D	d	f#	b	d#	g	b b	d	D	B b	F	D b	A	F#
E b	e b	g	c	e	a b	b	e b	E b	B	G b	D	B b	G
E	e	g#	c#	f	a	c	e	E	C	G	E b	B	A b
F	f	a	d	f#	b b	c#	f	F	D b	A b	E	C	A
F#	f#	b b	d#	g	b	d	f#	F#	D	A	F	D b	B b
G	g	b	e	g#	c	e b	g	G	E b	B b	G b	D	B
A b	a b	c	f	a	c#	e	g#	A b	E	B	G	E b	C
A	a	c#	f#	b b	d	f	a	A	F	C	A b	E	D b
B b	b b	d	g	b	e b	f#	b b	B b	G b	D b	A	F	D
B	b	d#	g#	c	e	g	b	B	G	D	B b	F#	E b

Each table is the other read backwards, since every move undoes itself. Enharmonic names are the same triad ($c\sharp = d\flat, g\sharp = a\flat, F\sharp = G\flat$).

Combining moves

Chains of P, L and R reach every major and minor triad. Read a chain left to right: PR means P, then R. On the grid a chain is a walk from triangle to triangle, crossing one edge per move.

Two moves Two flips bring the mode back, so every two-move chain lands on a triad of the same mode.

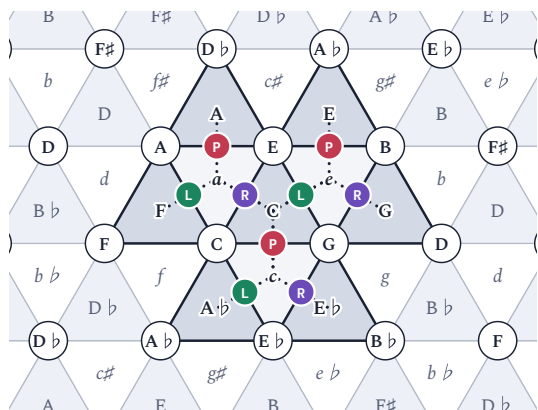
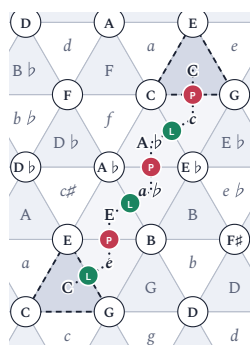


Figure 2. From C major, the six two-move chains reach the six major triads that surround it.

CHAIN	FROM C MAJOR	IN C	RELATION	HOLDS	FROM C MINOR
L R	$C \rightarrow e \rightarrow G$	V	dominant	G	$c \rightarrow A \flat \rightarrow f$
R L	$C \rightarrow a \rightarrow F$	IV	subdominant	C	$c \rightarrow E \flat \rightarrow g$
P R	$C \rightarrow c \rightarrow E \flat$	$\flat$ III	chromatic mediant	G	$c \rightarrow C \rightarrow a$
P L	$C \rightarrow c \rightarrow A \flat$	$\flat$ VI	chromatic mediant	C	$c \rightarrow C \rightarrow e$
L P	$C \rightarrow e \rightarrow E$	III	chromatic mediant	E	$c \rightarrow A \flat \rightarrow a \flat$
R P	$C \rightarrow a \rightarrow A$	VI	chromatic mediant	E	$c \rightarrow E \flat \rightarrow e \flat$

Chromatic mediants are same-mode triads with roots a third apart that share one note. Their sudden shift of color is common in 19th-century music and film scores.

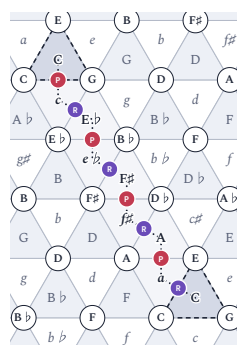
Cycles Alternate two moves and you eventually return to the start. On the flat grid the walk runs along a strip and ends on another copy of the starting triad.



P L Hexatonic cycle

$C \rightarrow c \rightarrow A \flat \rightarrow a \flat \rightarrow E \rightarrow e \rightarrow C$

Six triads, each sharing two notes with the next. Together they use only six pitch classes, the **hexatonic scale** $C E \flat E G A \flat B$ (half step and minor third, alternating). There are four such cycles. Each triad's hexatonic pole (H) sits halfway round.

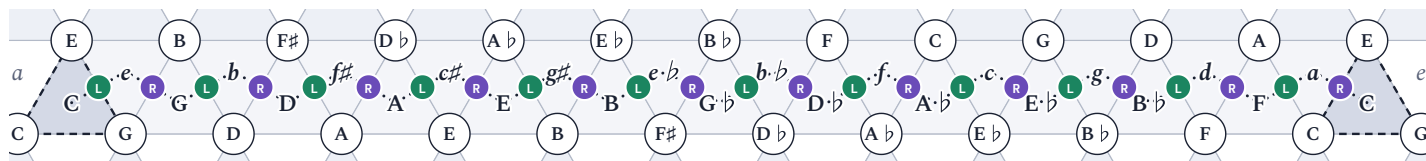


P R Octatonic cycle

$C \rightarrow c \rightarrow E \flat \rightarrow e \flat \rightarrow F \sharp \rightarrow f \sharp \rightarrow A \rightarrow a \rightarrow C$

Eight triads; the major roots rise by minor thirds (C, E $\flat$, F $\sharp$, A). Together they fill the **octatonic scale** $C D \flat E \flat F \sharp G A B \flat$ (half step and whole step, alternating). Three such cycles share out all 24 triads.

L R LR cycle: all 24 triads



$C \rightarrow e \rightarrow G \rightarrow b \rightarrow D \rightarrow f \sharp \rightarrow A \rightarrow c \sharp \rightarrow E \rightarrow g \sharp \rightarrow B \rightarrow e \flat \rightarrow G \flat \rightarrow b \flat \rightarrow D \flat \rightarrow f \rightarrow A \flat \rightarrow c \rightarrow E \flat \rightarrow g \rightarrow B \flat \rightarrow d \rightarrow F \rightarrow a \rightarrow C$

Alternating L and R visits every major and minor triad once. The major triads climb the circle of fifths (C, G, D, A, ...), and each minor triad in between shares two notes with the triads on either side. The names switch from sharps to flats halfway round ($d \sharp = e \flat$).

How the moves fit together

The moves mirror with the mode. R raises a major triad's fifth by a whole step but lowers a minor triad's root by a whole step. The letter names a relationship, so $C \rightarrow a$ and $a \rightarrow C$ are both R.

Every triad is reachable. P, L and R together reach all 24 major and minor triads. They generate a group of 24 operations, the dihedral group of order 24, which mirrors the transpositions and inversions of pitch-class set theory.

Smoothness. P and L move one voice a half step (1 semitone in all). R moves one voice a whole step, and S and N move two voices a half step each (2 in all). H moves all three voices (3).

Watch the German names. In German, *Parallele* means the relative key, and the same-root key is the *Variante*. The English letters follow Anglo-American usage: P for parallel ($C \leftrightarrow c$), R for relative ($C \leftrightarrow a$).

The triad network

This page turns the Tonnetz inside out. Each triad becomes a single point, and an arrow joins two triads that are one move apart. Every triad has exactly three arrows, one each for P, L and R, so the points link into a honeycomb. Jack Doughett and Peter Steinbach (1998) called this layout the chicken-wire torus.

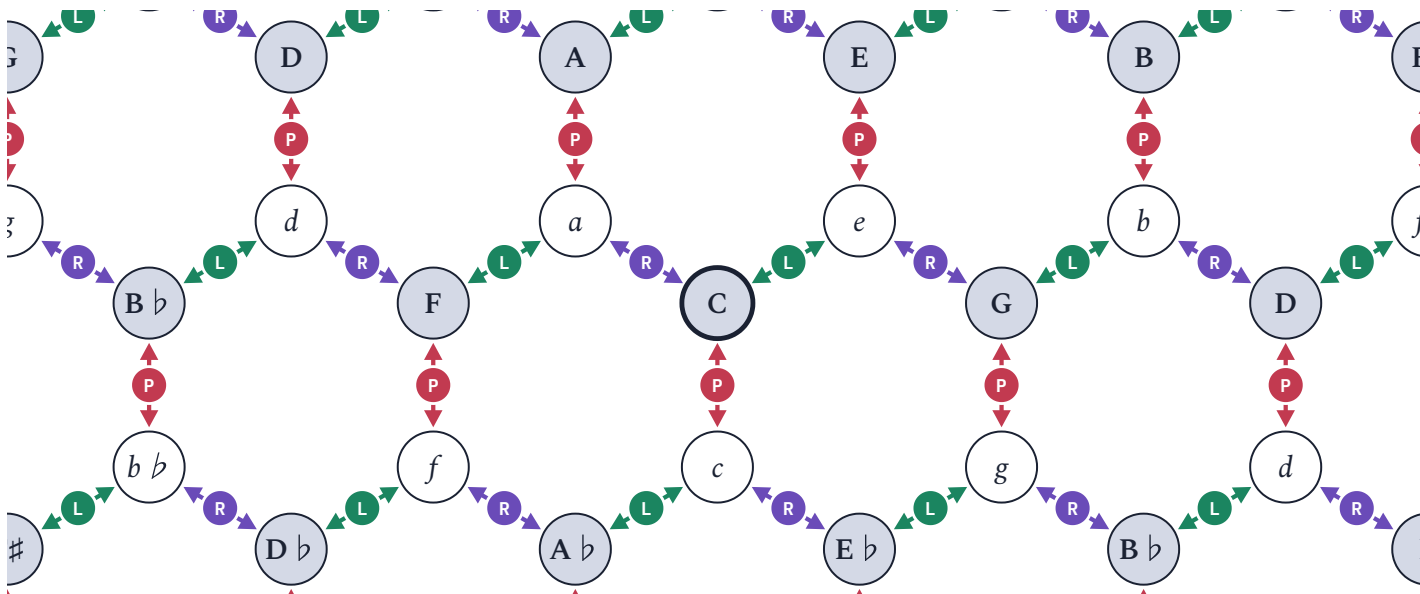


Figure 3. The triad network around C major (outlined). Shaded circles with capitals are major triads, and white circles with lower case are minor. Arrows point both ways because every move undoes itself.

Same map, inside out.

A triangle on the Tonnetz is a point here, an edge two triangles share is an arrow, and a note is a hexagon of the six triads that contain it.

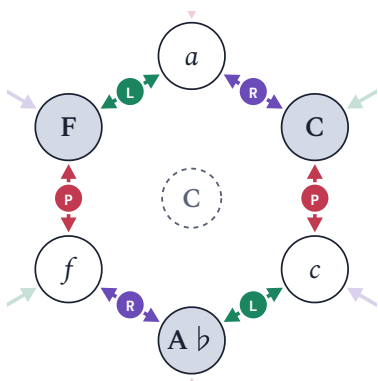
Direction tells you the move.

From a major triad, P points down, L up and to the right, and R up and to the left. From a minor triad each one points the opposite way.

Count arrows for distance.

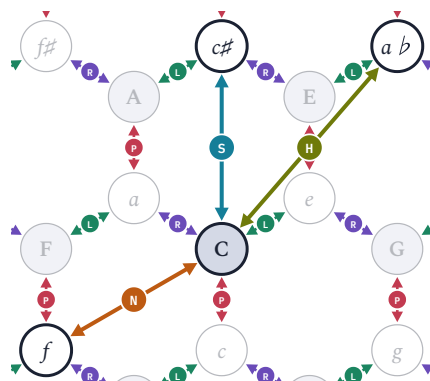
The shortest path between two triads is the fewest moves between them. From C, three triads are one arrow away and six are two. None is more than five; *b b* minor is the farthest.

Patterns to look for



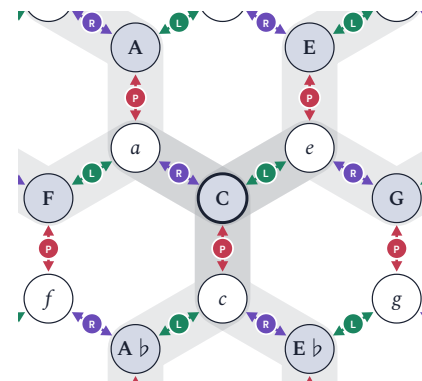
One note, one hexagon

The six triads that contain C ring a hexagon. Walking round it alternates R, L and P: C, *a*, F, *f*, *Ab*, *c*, and back to C. All six keep C as a common tone.



S N H Compound jumps

S and N cross a hexagon to its opposite corner, so the two triads share only the note that hexagon stands for: E for C → *c* #, C for C → *f*. H crosses three arrows and shares nothing.



Cycles run straight

Alternating two moves walks a straight zigzag (shaded). LR runs side to side through all 24 triads. PL runs on the ↗ diagonal and PR on the ↘ diagonal, through six and eight triads before the pattern repeats.